4.3 Differentiation of Parametric Equations

Question Paper

Course	CIE A Level Maths
Section	4. Differentiation
Topic	4.3 Differentiation of Parametric Equations
Difficulty	Hard

Time allowed: 60

Score: /45

Percentage: /100

Question 1a

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t for the parametric equations

$$x = \sin 2t \qquad\qquad y = e^t$$

[3 marks]

Question 1b

(b) Verify that the graph of x against y passes through the point $(0\ ,1)$ and find the gradient at that point.

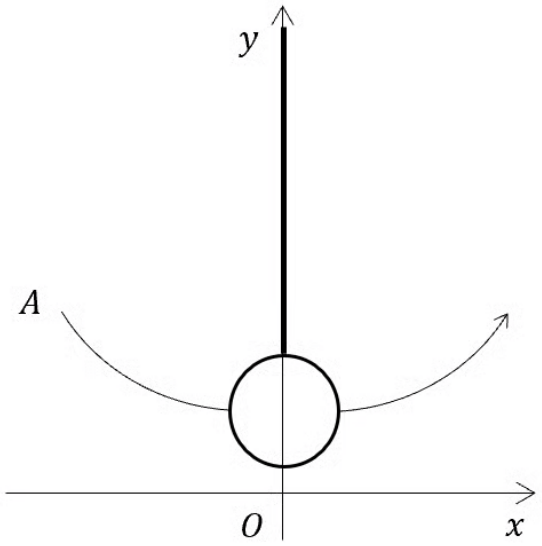
[2 marks]

Question 2a

A crane swings a wrecking ball along a two-dimensional path defined by the parametric equations

$$x = 8t - 4 \qquad y = 16t^2 - 16t + 5 \qquad 0 \leq t \leq 1$$

as shown in the diagram below.



x and y are, respectively, the horizontal and vertical displacements in metres from the origin, O, and t is the time in seconds. Point A indicates the initial position of the wrecking ball, at time $t = 0$.

(a) Find a Cartesian equation of the curve in the form $y = \mathrm{f}(x)$, and state the domain of $\mathrm{f}(x)$.

[3 marks]

Question 2b

(b) Find the difference between the maximum and minimum heights of the wrecking ball during its motion.

[2 marks]

Question 2c

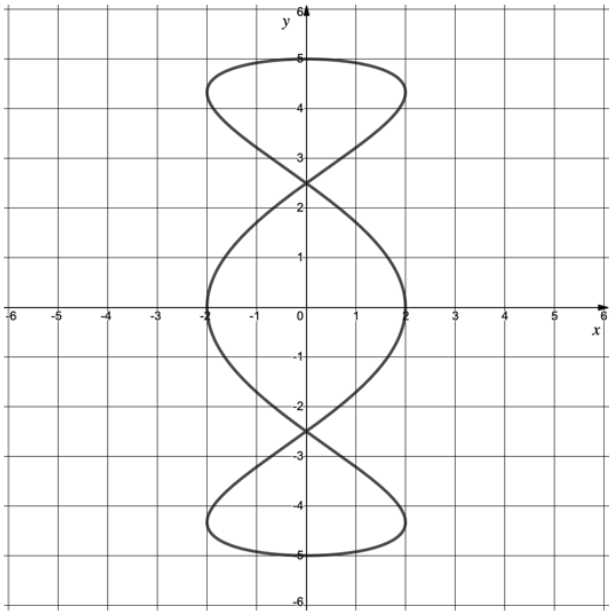
(c) The crane is positioned such that point A is 7 m horizontally from the wall the wrecking ball is to destroy.
Find the height at which the wrecking ball will strike the wall.

[3 marks]

Question 3a

The graph of the curve C shown below is defined by the parametric equations

$$x = 2\cos 3\theta \qquad y = 5\sin\theta \qquad 0 \leq \theta \leq 2\pi$$



(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ.

[3 marks]

Question 3b

(b) (i) Show that the gradient of the tangent to C, at the point where $\theta = \frac{\pi}{4}$, is $-\frac{5}{6}$.

(ii) Hence find the equation of the tangent to C at the point where $\theta = \frac{\pi}{4}$.

[4 marks]

Question 4a

The curve C has parametric equations

$$x = \frac{1}{t^2} \qquad y = t + \frac{1}{t} \qquad t > 0$$

(a) Find an expression, in terms of t, for $\frac{dy}{dx}$.

[3 marks]

Question 4b

(b) (i) Find the gradient of the tangent to C at the point where $t = \frac{1}{2}$.

(ii) Hence find the equation of the normal to C at the point where $t = \frac{1}{2}$.

[5 marks]

Question 5a

The curve C has parametric equations

$$x = t^2 - 4 \qquad y = 3t$$

(a) Show that at the point $(0, 6)$, $t = 2$ and find the value of $\frac{dy}{dx}$ at this point.

[4 marks]

Question 5b

(b) The tangent at the point $(0\ ,6)$ is parallel to the normal at the point P.
Find the exact coordinates of point P

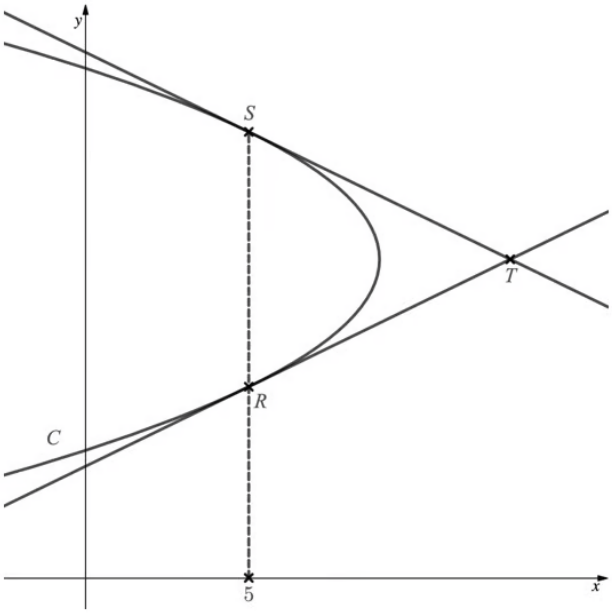
[3 marks]

Question 6

A curve C has parametric equations

$$x = 9 - t^2 \qquad y = 5 - t$$

The tangents to C at the points R and S meet at the point T, as shown in the diagram below.



Given that the x-coordinate of both points R and S is 5, find the area of the triangle RST.

[10 marks]